

题号	5.2节 P88	答 题 内 容	廖国勇
- 1		$\int \frac{1}{\sqrt{3x+2}} dx = \frac{1}{3} \int (3x+2)^{-\frac{1}{2}} d(3x+2) = \frac{1}{3} \cdot \frac{1}{-\frac{1}{2}+1} (3x+2)^{-\frac{1}{2}+1} + C$ $= \frac{2}{3} \sqrt{3x+2} + C$	
2		$\int f'(2x) dx = \frac{1}{2} \int f'(2x) d(2x) = \frac{1}{2} f(2x) + C$	
3		$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d(\ln x) = \ln \ln x + C$ 注 $\frac{1}{x} dx = d(\ln x) = d \ln x$, 本题中含有 $\ln x$, 说明 $x > 0$	
4		$\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int (1-x^2)^{-\frac{1}{2}} d(1-x^2) = -\frac{1}{2} \cdot \frac{1}{1+(-\frac{1}{2})} (1-x^2)^{-\frac{1}{2}+1} + C$ $= -\sqrt{1-x^2} + C$	
二 1		求不定积分 $\int \frac{1}{1+e^x} dx$ 解: 原式 $= \int \frac{e^x}{(1+e^x)e^x} dx = \int \frac{1}{e^x(1+e^x)} de^x$ $= \int \left(\frac{1}{e^x} - \frac{1}{1+e^x} \right) de^x = \int \frac{1}{e^x} de^x - \int \frac{1}{1+e^x} d(1+e^x)$ $= \ln e^x - \ln(1+e^x) + C = x - \ln(1+e^x) + C$ 注, 1. 答案为 $x - \ln 1+e^x + C$, 其实 $1+e^x > 0$, 绝对值符号可以不要。 2. 不定积分解法很多, 下面举另一解法。 另解: 设 $t = e^x$, 则 $x = \ln t$, $dx = \frac{1}{t} dt$, 于是 $\text{原式} = \int \frac{1}{1+t} \cdot \frac{1}{t} dt$ $= \int \left(\frac{1}{t} - \frac{1}{1+t} \right) dt = \int \frac{1}{t} dt - \int \frac{1}{1+t} d(1+t)$ $= \ln t + \ln 1+t + C$ $= x + \ln(1+e^x) + C \quad (\text{回代})$	

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二 2 求不定积分 $\int (\cos x + \cos^5 x) dx$

$$\begin{aligned}
 \text{解: 原式} &= \int (1 + \cos^4 x) \cos x dx = \int [1 + (1 - \sin^2 x)] d\sin x \\
 &= \int (2 - 2\sin^2 x + \sin^4 x) d\sin x \\
 &= 2\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C
 \end{aligned}$$

P89

3 求不定积分 $\int \frac{1}{x^2(1-x^2)} dx$

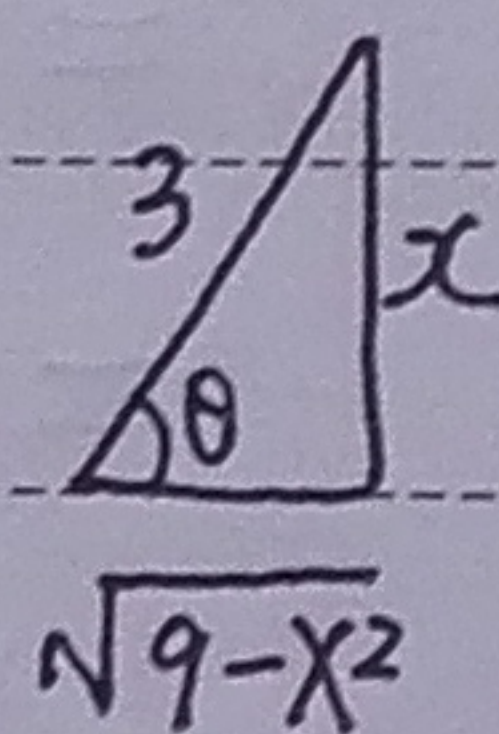
$$\begin{aligned}
 \text{解: 原式} &= \int \left(\frac{1}{x^2} + \frac{1}{1-x^2} \right) dx = \int \left[\frac{1}{x^2} + \frac{1}{2} \left(\frac{1}{1-x} + \frac{1}{1+x} \right) \right] dx \\
 &= \int \frac{1}{x^2} dx + \frac{1}{2} \int \frac{1}{1-x} dx + \frac{1}{2} \int \frac{1}{1+x} dx \\
 &= -\frac{1}{x} - \frac{1}{2} \ln|1-x| + \frac{1}{2} \ln|1+x| + C \\
 &= -\frac{1}{x} + \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C
 \end{aligned}$$

4 求不定积分 $\int \frac{1-x}{\sqrt{9-x^2}} dx$ 解: 设 $x = 3\sin\theta$, 则 $dx = 3\cos\theta d\theta$, $\sqrt{9-x^2} = 3\cos\theta$ 于是, 原式 $= \int \frac{1-3\sin\theta}{3\cos\theta} 3\cos\theta d\theta$

$$= \int (1 - 3\sin\theta) d\theta$$

$$= \theta + 3\cos\theta + C$$

$$= \arcsin \frac{x}{3} + \sqrt{9-x^2} + C$$

5 求不定积分 $\int \frac{\sin^2 x}{1+\sin^2 x} dx$

$$\text{解: 原式} = \int \left(1 - \frac{1}{1+\sin^2 x} \right) dx = x - \int \frac{1}{\cos^2 x + 2\sin^2 x} dx$$

$$= x - \int \frac{1}{1+2\tan^2 x} \cdot \frac{1}{\cos^2 x} dx$$

$$= x - \int \frac{1}{1+2\tan^2 x} d(\tan x) = x - \int \frac{\frac{1}{\sqrt{2}}}{1+(\sqrt{2}\tan x)^2} d(\sqrt{2}\tan x)$$

$$= x - \frac{1}{\sqrt{2}} \arctan(\sqrt{2}\tan x) + C$$

$$\text{注 } \frac{1}{\cos^2 x} dx = \sec^2 x dx = d(\tan x) \quad \int \frac{1}{1+u^2} du = \arctan u + C$$

二 6 求不定积分 $\int \frac{x^2}{\sqrt{1-x^2}} dx$

解: 设 $x = \sin \theta$, 则 $\sqrt{1-x^2} = \cos \theta$, $dx = \cos \theta d\theta$

$$\text{原式} = \int \frac{\sin^2 \theta}{\cos \theta} \cos \theta d\theta = \int \sin^2 \theta d\theta$$

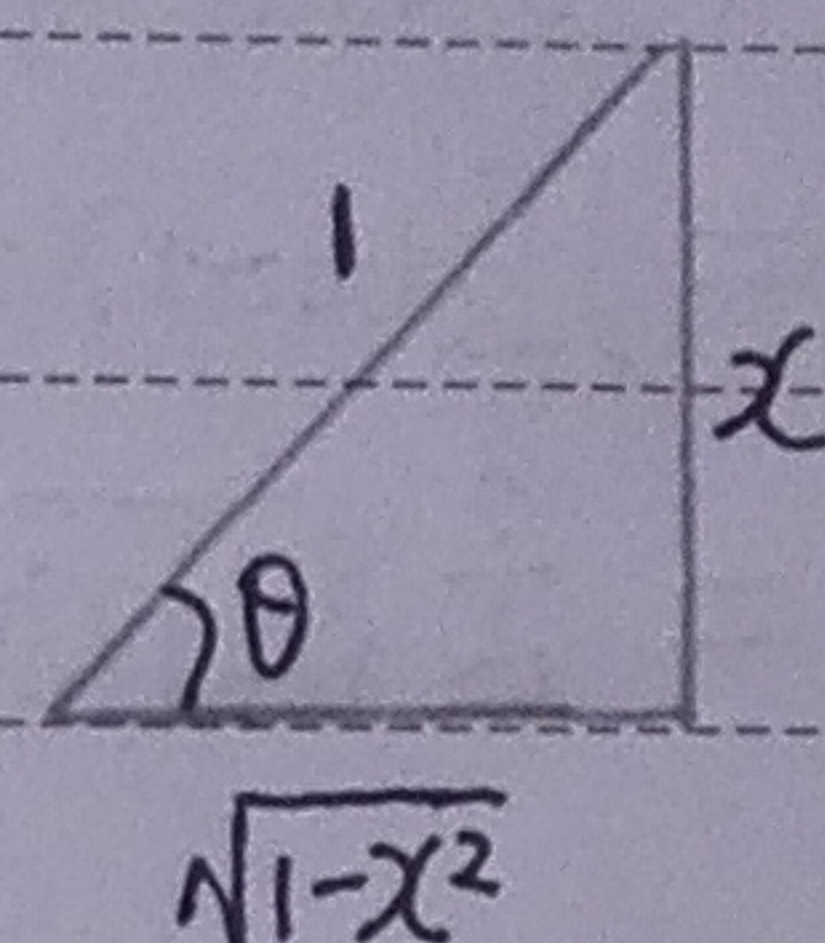
$$= \int \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \int \frac{1}{2} d\theta - \frac{1}{4} \int \cos 2\theta d(2\theta)$$

$$= \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta + C$$

$$= \frac{1}{2} \arcsin x - \frac{1}{4} x \cdot \sqrt{1-x^2} \cdot 2 + C$$

$$= \frac{1}{2} \arcsin x - \frac{1}{2} x \sqrt{1-x^2} + C$$



注 $\sin 2\theta = 2 \sin \theta \cos \theta = 2 \cdot x \cdot \sqrt{1-x^2}$

7 求不定积分 $\int \frac{1}{x^2 \sqrt{1+x^2}} dx$

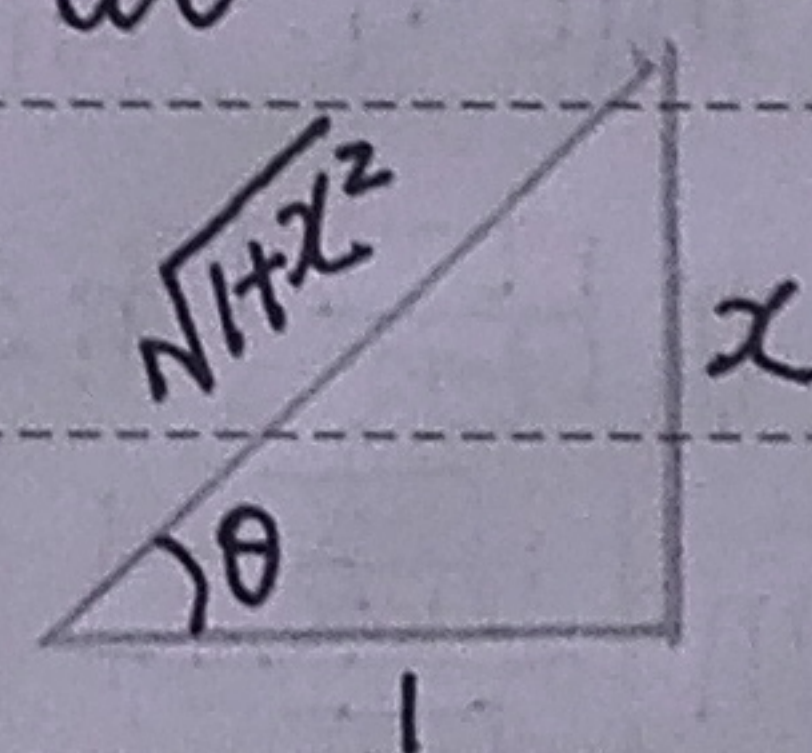
解: 设 $x = \tan \theta$, 则 $\sqrt{1+x^2} = \sec \theta$, $dx = \sec^2 \theta d\theta$

$$\text{原式} = \int \frac{1}{\tan^2 \theta \cdot \sec \theta} \sec^2 \theta d\theta = \int \frac{\cos \theta}{\sin^2 \theta} d\theta$$

$$= \int \sin^2 \theta d \sin \theta$$

$$= -\frac{1}{\sin \theta} + C$$

$$= -\frac{\sqrt{1+x^2}}{x} + C$$



8 求不定积分 $\int \frac{\sqrt{x^2-4}}{x} dx$

解: 设 $x = 2 \sec \theta$, 则 $dx = 2 \sec \theta \tan \theta d\theta$

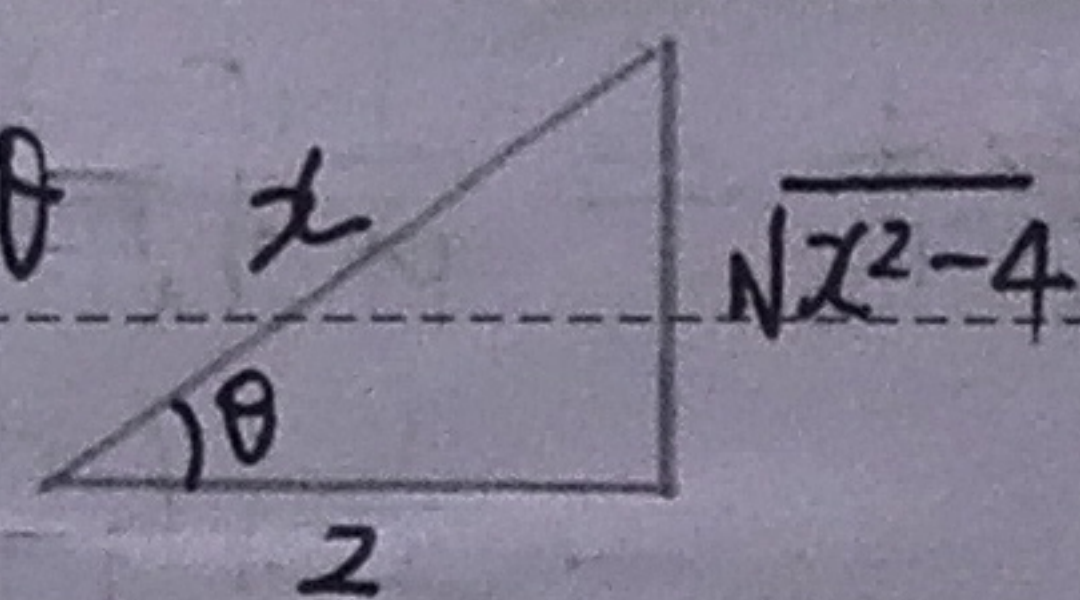
$\sqrt{x^2-4} = 2 \tan \theta$, 于是

$$\text{原式} = \int \frac{2 \tan \theta}{2 \sec \theta} 2 \sec \theta \tan \theta d\theta = \int 2 \tan^2 \theta d\theta$$

$$= 2 \int (\sec^2 \theta - 1) d\theta = 2 \tan \theta - 2\theta + C$$

$$= \sqrt{x^2-4} - 2 \arcsin \frac{\sqrt{x^2-4}}{x} + C$$

注 $\theta = \arccos \frac{2}{x} = \arcsin \frac{\sqrt{x^2-4}}{x} = \arctan \frac{\sqrt{x^2-4}}{2} = \operatorname{arccot} \frac{2}{\sqrt{x^2-4}}$



二 9 求不定积分 $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解: 设 $t = \sqrt[3]{1+x}$, 则 $x = t^3 - 1$, $dx = 3t^2 dt$, 于是

$$\text{原式} = \int \frac{1}{1+t} 3t^2 dt = \int \left(\frac{3t^2-3}{1+t} + \frac{3}{1+t} \right) dt$$

$$= \int \left[3(t-1) + \frac{3}{1+t} \right] dt = \frac{3}{2}t^2 - 3t + 3 \ln|1+t| + C$$

$$= \frac{3}{2} \sqrt[3]{1+x}^2 - 3 \sqrt[3]{1+x} + 3 \ln|1+\sqrt[3]{1+x}| + C$$

10 求不定积分 $\int \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx$

解: 设 $t = \sqrt{\frac{1-x}{1+x}}$ 则 $x = \frac{1-t^2}{1+t^2}$ $dx = \frac{-2t(1+t^2) - 2t(1-t^2)}{(1+t^2)^2} dt$

$$= \frac{-4t}{(1+t^2)^2}, \text{ 于是有}$$

$$\text{原式} = \int \left(\frac{1-t^2}{1+t^2} \right)^{-1} \cdot t \cdot \frac{-4t}{(1+t^2)^2} dt = \int \frac{4t^2}{t^4-1} dt$$

$$= \int 2 \left(\frac{1}{t^2-1} + \frac{1}{t^2+1} \right) dt$$

$$= \int \left(\frac{1}{t-1} - \frac{1}{t+1} + \frac{2}{t^2+1} \right) dt$$

$$= \ln|t-1| - \ln|t+1| + 2 \arctan t + C$$

$$= \ln \left| \frac{t-1}{t+1} \right| + 2 \arctan t + C$$

$$= \ln \left| \frac{\sqrt{\frac{1-x}{1+x}} - 1}{\sqrt{\frac{1-x}{1+x}} + 1} \right| + 2 \arctan \sqrt{\frac{1-x}{1+x}} + C$$

$$= \ln \left| \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} + \sqrt{1+x}} \right| + 2 \arctan \sqrt{\frac{1-x}{1+x}} + C$$

11 求不定积分 $\int \frac{x^3}{\sqrt{1+x^2}} dx$

解: 原式 = $\frac{1}{2} \int \frac{x^2}{\sqrt{1+x^2}} d(1+x^2) = \frac{1}{2} \int \left(\frac{1+x^2}{\sqrt{1+x^2}} - \frac{1}{\sqrt{1+x^2}} \right) d(1+x^2)$

$$= \frac{1}{2} \int \sqrt{1+x^2} d(1+x^2) - \frac{1}{2} \int \frac{1}{\sqrt{1+x^2}} d(1+x^2)$$

$$= \frac{1}{2} \cdot \frac{2}{3} (1+x^2)^{\frac{3}{2}} - \frac{1}{2} \cdot \frac{1}{-\frac{1}{2}+1} (1+x^2)^{-\frac{1}{2}+1} + C$$

$$= \frac{1}{3} (1+x^2)^{\frac{3}{2}} - (1+x^2)^{\frac{1}{2}} + C$$

题号

5.2节 P92

答题内容

廖国勇

二 12

求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

$$\text{解: 原式} = \int \frac{e^x}{\sqrt{1+e^x} \cdot e^x} dx = \int \frac{1}{\sqrt{1+e^x}} e^x dx$$

$$= \int \left(\frac{\sqrt{1+e^x}}{e^x} - \frac{1}{\sqrt{1+e^x}} \right) dx$$

设 $t = \sqrt{1+e^x}$, 则 $x = \ln(t^2 - 1)$ $dx = \frac{2t}{t^2 - 1} dt$, 于是

$$\text{原式} = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \left(\frac{1}{t-1} - \frac{1}{t+1} \right) dt$$

$$= \ln \left| \frac{t-1}{t+1} \right| + C = \ln \frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1} + C$$

13 求不定积分 $\int \frac{1}{(2x^2+1)\sqrt{1+x^2}} dx$

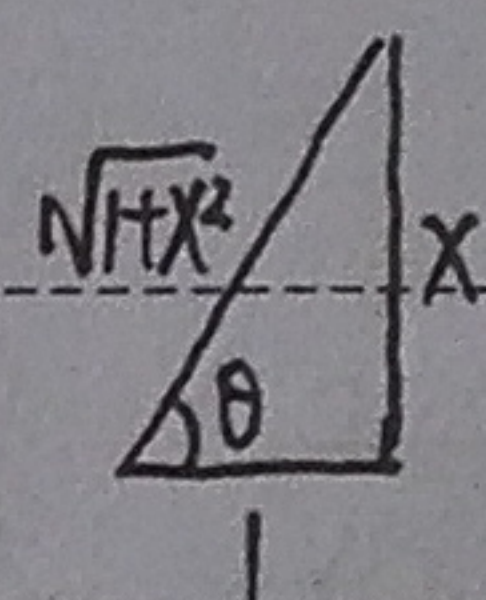
解: 设 $x = \tan \theta$, 则 $\sqrt{1+x^2} = \sec \theta$, $dx = \sec^2 \theta d\theta$

$$\text{原式} = \int \frac{1}{(2\tan^2 \theta + 1) \cdot \sec \theta} \sec^2 \theta d\theta$$

$$= \int \frac{\cos \theta}{2\sin^2 \theta + \cos^2 \theta} d\theta = \int \frac{1}{1 + \sin^2 \theta} d\sin \theta$$

$$= \arctan \sin \theta + C$$

$$= \arctan \frac{x}{\sqrt{1+x^2}} + C$$

14 求不定积分 ~~$\int \frac{1}{x\sqrt{1+x^2}} dx$~~ $\int \frac{1}{x + \sqrt{1-x^2}} dx$

解: 设 $x = \sin \theta$, 则 $\sqrt{1-x^2} = \cos \theta$, $dx = \cos \theta d\theta$, 于是

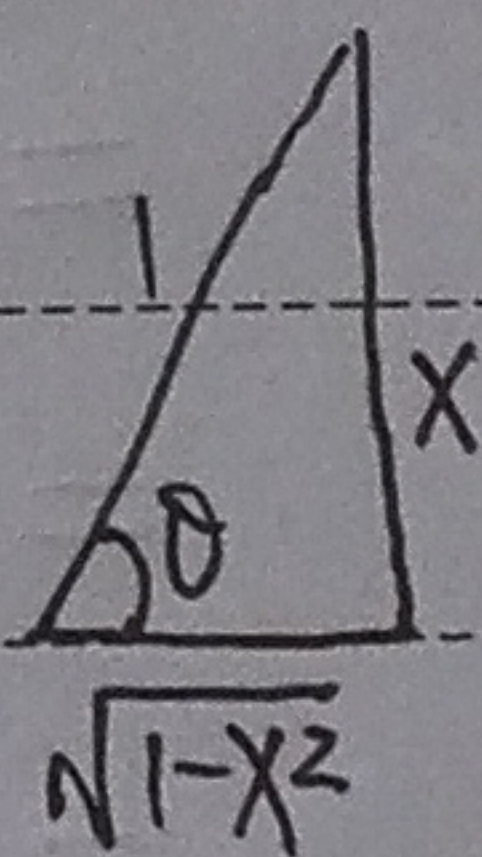
$$\text{原式} = \int \frac{1}{\sin \theta + \cos \theta} \cos \theta d\theta$$

$$= \int \frac{\frac{1}{2}(\sin \theta + \cos \theta) + (\cos \theta \sin \theta)}{\sin \theta + \cos \theta} d\theta$$

$$= \frac{1}{2} \theta - \frac{1}{2} \int \frac{(\sin \theta + \cos \theta)'}{\sin \theta + \cos \theta} d\theta$$

$$= \frac{1}{2} \theta - \frac{1}{2} \ln |\sin \theta + \cos \theta| + C$$

$$= \frac{1}{2} \arcsin x + \frac{1}{2} \ln |x + \sqrt{1-x^2}| + C$$



15 求不定积分 $\int \sin^4 x dx$

$$\begin{aligned} \text{解: 原式} &= \int \left(\frac{1-\cos 2x}{2} \right)^2 dx = \int \frac{1}{4} (1-2\cos 2x + \cos^2 2x) dx \\ &= \int \frac{1}{4} \left[1-2\cos 2x + \frac{1+\cos 4x}{2} \right] dx \\ &= \int \left(\frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x \right) dx \\ &= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C \end{aligned}$$

16 求不定积分 $\int \frac{\cos x}{\sin x + \cos x} dx$

$$\begin{aligned} \text{解: 原式} &= \int \frac{1}{2} \frac{(\sin x + \cos x) + (\sin x + \cos x)'}{\sin x + \cos x} dx \\ &= \frac{1}{2} \int dx + \frac{1}{2} \int \frac{1}{\sin x + \cos x} d(\sin x + \cos x) \\ &= \frac{1}{2}x + \frac{1}{2} \ln |\sin x + \cos x| + C \end{aligned}$$

17 求不定积分 $\int \frac{1}{x\sqrt{1+x^4}} dx$

解: 设 $\sqrt{1+x^4} = u$, 则 $u^2 = 1+x^4$, $2udu = 4x^3 dx$, 于是

$$\begin{aligned} \text{原式} &= \int \frac{\frac{1}{4}}{x^4 \sqrt{1+x^4}} 4x^3 dx \\ &= \int \frac{\frac{1}{4}}{(u^2-1)u} \cdot 2u du = \frac{1}{2} \int \frac{1}{u^2-1} du \\ &= \frac{1}{4} \int \left(\frac{1}{u-1} + \frac{-1}{u+1} \right) du \\ &= \frac{1}{4} \ln \left| \frac{u-1}{u+1} \right| + C \\ &= \frac{1}{4} \ln \frac{\sqrt{1+x^4}-1}{\sqrt{1+x^4}+1} + C \\ &= \frac{1}{4} \ln \frac{(\sqrt{1+x^4}-1)(\sqrt{1+x^4}+1)}{(\sqrt{1+x^4}+1)(\sqrt{1+x^4}+1)} + C \\ &= \frac{1}{4} \ln x^4 - \frac{1}{2} \ln(1+\sqrt{1+x^4}) + C \\ &= \ln|x| - \frac{1}{2} \ln(1+\sqrt{1+x^4}) + C \end{aligned}$$